NUCLEAR CHARGE RADII AND ELECTRIC QUADRUPOLE MOMENTS OF EVEN-EVEN ISOTOPES *

Bożena Nerlo-Pomorska and Beata Mach

Department of Theoretical Physics, Maria Curie-Skłodowska University,

PL 20-031 Lublin, Poland

Isotope shifts of the mean square radii (MSR) and electric quadrupole moments of even-even nuclei with $20 \leq Z \leq 98$ are calculated using a dynamical microscopic model.

A single particle Nilsson potential with the Seo set of correction term parameters, the pairing forces in the BCS formalism and a long range interaction in the local approximation are used. A collective hamiltonian is obtained using a generator coordinate method (GCM) with the gaussian overlap approximation (GOA). A potential energy of the nucleus consists of a microscopic-macroscopic Strutinsky energy and a zero point vibrational term. A liquid droplet model is used as the macroscopic part of the potential. A BCS wave function is taken as a generator function and two collective variables, quadrupole and hexadecapole deformations, serve as the generator coordinates.

In general, a good agreement between the theory and experimental data is achieved when isospin dependence of the harmonic oscillator frequency, $\hbar\omega_0 = 40 \cdot A^{-1/3}$ MeV, is omitted. It has also been proven that the nuclear charge radius depends less on the neutron number N than it was previously thought. A new formula for the charge radius is proposed: $R_0 = 1.25\,(1 - 0.2 \cdot \frac{N-Z}{A}) A^{1/3} fm$.

* This work has been supported in part by the Polish Committee of Scientific Research under Contract No. 20311901.

CONTENTS

1. INTRODUCTION 3
2. MICROSCOPIC MODEL 4
3. RESULTS 5
   3.1 Light nuclei with $20 \leq Z \leq 48$, $Z \leq N \leq 40$ ($A_{av} \sim 44$) 7
   3.2 Neutron-rich and -deficient nuclei with $38 \leq Z \leq 74$, $Z \leq N \leq 74$ ($A_{av} \sim 100$) 7
   3.3 Rare-earth nuclei with $52 \leq Z \leq 78$, $80 \leq N \leq 120$ ($A_{av} \sim 165$) 8
   3.4 Actinides nuclei $80 \leq Z \leq 98$, $102 \leq N \leq 154$ ($A_{av} \sim 247$) 9
4. ISOSPIN DEPENDENCE OF THE NUCLEAR CHARGE RADIUS 9
5. CONCLUSIONS 13
6. REFERENCES 14
7. FIGURES CAPTIONS 15
8. EXPLANATIONS OF TABLES 16
9. TABLE 4 17
10. TABLE 5 20
11. TABLE 6 29
12. TABLE 7 43

1. INTRODUCTION

Thanks to a development of laser spectroscopic methods and the on-line mass separators a broad information on the ground state properties of nuclei, even those far from the β-stability line [1], has been obtained recently. Systematics of the quadrupole moments, Q_2, [2] and the isotopic or isotonic shifts of the mean square radii (MSR) of nuclei should supply good information on the general features of nuclear structure, especially concerning the size and shape of a nucleus. The mean square charge radius of a nucleus grows with the neutron number N due to the volume increase and also that due to a change in the deformation of a nucleus (shell effects). Unfortunately neither a macroscopic nor a microscopic model can describe simultaneously all the interesting features of the mean square charge radii systematics as for example: kink effects at the magic numbers, shape staggering between odd and even isotopes or parabolic dependence on N of the MSR isotope shifts in the Ca isotopes. As a general feature even for the spherical nuclei, the theoretical mean square radii grow faster with neutron number N than the experimental results. This issue was solved in Ref. [3] by removing the isospin dependence from the harmonic oscillator frequency, $\hbar\omega_0$, of the Nilsson single particle potential. Dynamical calculations of the quadrupole moments and mean square charge radii, which were performed in Ref. [3] using the microscopic collective model, give a nice agreement with the experimental data for nuclei with $38 \leq Z < 74$, $N \leq 74$.

A similar study of the same nuclear region, using a macroscopic model was performed in Ref. [4], where an isospin dependence of the liquid drop radius has been found. In particular, instead of the frequently used formula for the charge radius:

$$R_0 = 1.2 \cdot A^{1/3}\, fm \tag{1}$$

a new one was proposed [4]:

$$R_0 = 1.2\ [1 - 0.205 \cdot (I - I_\beta)]\, A^{1/3}\, fm\ , \tag{2}$$

where $I = (N - Z)/A$ is the reduced isospin of a nucleus and I_β is the isospin value of a β-stable nucleus with the same mass number A. The macroscopic model, which includes this new isospin dependence of a (sharp) charge radius, reproduces surprisingly well the

proper experimental isotope shifts of the MSR. This leads to a question of whether this new isospin dependence of a nuclear radius represents a general property for other nuclear regions as well.

It is the purpose of this work to extend studies done in Refs. [3] and [4] to all regions of nuclei known experimentally, to find the optimal parameters $\hbar\omega_0$ and R_0, and to predict the charge MSR and quadrupole moments for a range of nuclei which had not yet been investigated experimentally. Since here there is almost no further theoretical progress in the microscopic model beyond the one described in Ref. [3], only the main points of the theory are outlined in Section 2. In Section 3 the quadrupole moments and the MSR isotope shifts are presented for all even-even nuclei known experimentally. The microscopic equilibrium deformations are used to calculate the macroscopic mean square radii before their fit to the experimental data is performed. The results leading to the optimal R_0 parameters for all even-even nuclei are presented in Section 4. Conclusions and proposals for further studies are gathered at the end of the paper. In Tables (4 - 7) we show the complete microscopic results for a large range of Z and N.

2. MICROSCOPIC MODEL

The aim of our calculation is to obtain the equilibrium deformations and multipole moments of nuclei using the generator coordinate method with the gaussian overlap approximation [5]. A two dimensional collective variable space, i.e. quadrupole ε and hexadecapole ε_4 deformation parameters, is used as generator coordinates $a = \{a_1, a_2\}=\{\varepsilon, \varepsilon_4\}$. A BCS wave function is taken as a generator function

$$|a\rangle = \prod_{\nu}(u_\nu + v_\nu \alpha^+_\nu \alpha^+_{-\nu})|0\rangle \ , \tag{3}$$

where $|0\rangle$ is the vacuum particle state, v^2_ν are the occupation probabilities that in a single particle state $|\nu\rangle$ a pair of nucleons exists, and $u^2_\nu = 1 - v^2_\nu$. The single particle states are the eigenfunctions of a single particle hamiltonian $\widehat{H}_\circ$ with a deformed harmonic oscillator Nilsson potential. A correction term in $\widehat{H}_\circ$ was taken in the form proposed by Seo [6] for a single average mass number A_{av} which is different for every nuclear region. The harmonic

oscillator frequency has the value:

$$\hbar\omega_0 = 40 \cdot A^{-1/3} MeV \ , \tag{4}$$

which reproduces the experimental MSR values[7].

The mean field hamiltonian $\widehat{H}$ consists of a Nilsson potential, pairing forces and the long-range two-body correlations in a local approximation [8]. A collective hamiltonian $\widehat{H}_{coll}$ obtained by GCM + GOA consists of the kinetic $\widehat{T}$ and potential V parts. The potential energy of a nucleus is

$$V = \langle a|\widehat{H}|a\rangle - E_\circ \ . \tag{5}$$

The first term is taken as a Strutinsky [9] shell correction energy with a liquid droplet macroscopic part

$$\langle a|\widehat{H}|a\rangle = E_{LD} + \Delta E_{SHELL} \tag{6}$$

while $E_\circ$ is the zero point vibration energy. The equilibrium deformations are obtained by minimizing $V(a) =$ minimum $\longrightarrow a_\circ$.

A collective wave function Φ of the whole nucleus is obtained by a diagonalization of the collective hamiltonian $\widehat{H}_{coll}$ in the two dimensional harmonic oscillator base. The dynamical values of the mean square radii are calculated as the integrals

$$\langle r^2\rangle^A = \int \Phi^*(a)\langle a|r^2|a\rangle\Phi(a)\, da \ . \tag{7}$$

The MSR isotope shifts are defined as

$$\delta\langle r^2\rangle^{A,A'} = \langle r^2\rangle^A - \langle r^2\rangle^{A'} \ . \tag{8}$$

The quadrupole moments are calculated according to

$$Q_2 = \left[\int \Phi^*(a)\langle a|\widehat{Q}_{20}^2|a\rangle\Phi(a)\, da\right]^{1/2} \tag{9}$$

in order to compare them with their values obtained from the measured B(E2) transition probabilities. The quadrupole moment operator is

$$\widehat{Q}_{20} = 2r^2 P_2(cos\theta) \ . \tag{10}$$

where r, θ are the single particle coordinates and P_λ are the Legendre polynomials.

One has to stress that such dynamical description of a nucleus gives results different to previous static calculations (where all the microscopic quantities were found at the point of equilibrium) only for those cases when the potential energy has a weak nonharmonic dependence on deformation, the mass parameters are not constant, and the $\langle a|\widehat{Q}^2_{20}|a\rangle$ values do not depend linearly on the deformation. The collective wave function may then have a maximum at a deformation point different from the minimum in the potential, and the whole potential surface influences the final result.

3. RESULTS

The calculations were performed for all even-even nuclei having proton number larger that Z = 20, covering five regions of deformed nuclei each having an average mass A_{av}. The average pairing strengths G for every region were taken from Ref. [10]. The single particle potential parameters are the same as in Ref. [6] i.e. $\kappa_0 = 0.021$, $\kappa_1 = 0.9$, $\gamma_0 = 0.62$. The space of the collective variables covers a grid of quadrupole ε and hexadecapole ε_4 deformations:

ε= -0.60, -0.55, . . ., 0.60

ε_4= -0.12, -0.08, . . ., 0.12

Table 1 lists the investigated regions.

TABLE 1

Parameters used in the present calculation.

Region:	Light nuclei	Neutron-rich	Neutron-deficient	Rare-earth	Actinides
A_{av}	44	100	126	165	217
Z or N	20–48	38–74	50–80	52–120	80–154
$G(\hbar\omega_0)$	0.26	0.26	0.29	0.275	0.284

3.1. Light nuclei with $20 \leq Z \leq 48, Z \leq N \leq 48$ (A_{av} ~44)

In this region there exist only a few experimental data of the MSR isotope shifts. Ca isotopes show an interesting parabolic behaviour between N=20 and N=28 — their mean square radius increases up to 0.7 fm^2 for N=24, and then decreases again. This is due to the shell effects. Unfortunately our model does not reproduce the calcium results well.

The isotope shifts of the MSR for the even Ca – Ni isotopes are presented in Fig. 1. The MSR values are related to the isotope (assigned the dashed line) with the mass number A'. Points (•) and crosses (+) denote the theoretical results, and the experimental data [1], respectively. While the slope of the MSR agrees well with the experimental data for all isotopes, the strange behaviour of the Ca and Cr isotopes is not reproduced in our model. However, it has to be born in mind that lighter nuclei are not sufficiently well described within the mean field shell model. This is also seen in Fig. 2 which illustrates the quadrupole moments for even-even Ca – Mo isotopes. The theoretical values (points) are generally larger than the experimental data (crosses) especially for the heavier elements where the average mass $A_{av} = 44$ taken for the Seo integrals is too small. It means that the equilibrium deformations found in our model are too small. It is possible to adjust the parameters in order to reproduce the experimental results selectively for one or two elements, but this would be against our underlying philosophy which is to retain a uniform model covering the whole nuclear region. We therefore prefer to exclude those light isotopes from further investigation due to these unexplained features.

3.2. Neutron-rich and -deficient nuclei with $38 \leq Z \leq 74, Z \leq N \leq 74$ ($A_{av} \sim 100$)

This region covers lighter even-even Sr – W isotopes. The results obtained in our model have been already described in Ref. [3]. Consequently, they are repeated here only for the reason of completeness.

Figure 3 illustrates isotope shifts of the mean square radii related to the A' mass numbers (their corresponding N numbers are marked by the dashed lines). With the exception of the light Sr isotopes, our theoretical results (points) agree well with the experimental data (crosses) [1,2,11] and a proper slope of the MSR values is also achieved. Sr isotopes show some interesting shell features making the MSR results hard to reproduce even within the Hartree-Fock methods. A subtle interplay between prolate and oblate minima in the potential energy demands an inclusion of a nonaxial γ degree of freedom. We are planing to do this in the near future using the Bohr model. A shortage of this variable is seen in the case of ^{82}Sr where a prolate shape (open circle) would give a better MSR value than a deeper oblate minimum. This is also seen in the dependence on N of

the quadrupole moments.

Figure 4 illustrates quadrupole moments of even-even Sr – Nd isotopes. The theoretical results (points) jump occasionally up or down (as in the case of ^{104}Mo or ^{106}Ru) and it is obvious that the other (prolate or oblate) minimum would be favoured by nature. A difference between deformation energies in such cases is very small, less than 0.5 MeV, and it is really uncertain which one should be chosen. A small change in the parameters can shift an equilibrium from a prolate to an oblate minimum. In any case, it is necessary to stress that the dynamical effects are much larger than the uncertainty in the results caused by the γ softness of a nucleus. The quadrupole moments of Sr and Zr isotopes are very well reproduced at $N \approx 60$ where a sudden growth of the quadrupole deformation is due to an equilibrium jump from an oblate to a prolate shape.

3.3. Rare-earth nuclei with $52 \leq Z \leq 78, 80 \leq N \leq 120$ (A$_{av}$ ∼165)

The heavier rare-earth nuclei are well deformed. They have a deep energy minima with a positive quadrupole deformation except for the spherical elements at $N \approx 80$ and the heaviest isotopes which are oblate. There still remains a problem in reproducing the theoretical kink of the MSR isotope shifts in the Xe and Ba isotopes around $N = 82$. A rapid growth of their experimental values is difficult to describe within our model, although the behaviour of the radii is well reproduced for the lighter and heavier elements. The magic shell structure around $N = 82$ demands a somewhat more sophisticated approach and we shall include pairing vibration and quadrupole pairing forces into our model in order to check their influence on the MSR. In any case, for the heavier elements of Ce – Pt our theoretical results presented in Fig. 5 (points) agree rather well with the experimental data (crosses). The $\delta\langle r^2\rangle^{A,A'}$ results are related to the A' mass numbers (their corresponding N values are marked by the dashed lines).

Figure 6 illustrates quadrupole moments for the rare-earth nuclei. The theoretical results (points) are found to be too small only for the lighter isotopes of Xe – Nd. The Q_2 moments of the heavier elements agree very well with the experimental data (crosses). Note, that the theoretical quadrupole moments are found again too small for the heaviest Pt isotopes, which is probably due to the fact that their mass numbers are much larger than $A_{av} = 165$ taken for this region. Also the masses of Xe and Ba are significantly smaller than A_{av} what can partly spoil the Q_2 estimates (Better results are presented in Fig. 4).

3.4. Actinides with $80 \leq Z \leq 98, \;\; 102 \leq N \leq 154 \;\; (A_{av} \sim 217)$

The lighter elements from the actinides region demand inclusion of the octupole ε_3 degree of freedom which is not done in the present calculation. However, as one can see in Fig. 7, the results for the MSR isotope shifts are quite satisfactory with the exception of ^{202}Pb for which the calculated quadrupole deformation is too large. The theoretical results (points) for the heaviest Pb and Rn isotopes are too small in comparison with the experimental data (crosses). In general, slopes of the theoretical results are smaller than those of the experimental systematics. This is due to the fact that here the experimental quadrupole moments (shown as crosses in Fig. 8) are not well reproduced in our model. We also note, that quadrupole deformations of the Hg and Rn isotopes are too small. The calculated Q_2 moments for Pb nuclei agree surprisingly well with experimental data (crosses). The shell structure at $N = 126$ causes a rapid growth of the quadrupole moments in Hg, Pb, Po and Rn. In the case of ^{208}Pb, the calculated quadrupole moment is in agreement with the experimental result. One might expect that for the heavier elements the inclusion of an octupole deformation in Ra, Th, U could improve the slopes of the theoretical quadrupole moments.

Despite some discrepancies, in general, our theoretical description of all even-even nuclei is reasonable good. In the following chapter we shall use the equilibrium deformations of all isotopes for which the MSR shifts are measured (excluding the light region of nuclei) to find the optimal dependence of the nuclear liquid drop charge radius on the neutron excess (see also Ref. [13]).

4. ISOSPIN DEPENDENCE OF THE NUCLEAR RADIUS.

The first attempt to get the explicit isospin dependence of the nuclear radius has already been taken in Ref. [4] for the case of the experimentally known even-even Sr – W isotopes ($N \leq 80$). In that study the nuclear radius was found to depend to a lesser extent on the neutron number than was assumed in the liquid drop model [see eqn.(1)]. We, now extend this investigation to all nuclei with Z $\geq$ 36, and fit the macroscopic (uniform charge distribution) MSR to all experimentally known results for the 220 even-even isotopes. The ground state deformations $(\varepsilon^0, \varepsilon_4^0)$ are obtained following a description given in Section 2. The MSR values are calculated using the macroscopic formula:

$$\langle r^2 \rangle = \int_{\mathcal{V}(\varepsilon^0, \varepsilon_4^0)} \varrho \, r^2 \, d\tau \; , \tag{11}$$

where

$$\varrho = \begin{cases} \frac{Ze}{\mathcal{V}}, & \text{for r} \leq \text{R} \\ 0, & \text{for r} > \text{R} \end{cases} \; . \tag{12}$$

$\mathcal{V}$ is the volume of the deformed nucleus, R it's deformation dependent radius. We are looking for the best set of the α and p parameters in the formula of the corresponding spherical nuclear radius:

$$R_0 = r_0 \left[1 - \alpha \cdot \left(\frac{N-Z}{A}\right)^p\right] A^{1/3} \ . \tag{13}$$

The parameter r_0 is fixed from the experimental value of the radius for ^{197}Au.

As only the isotope shifts of the MSR values are measured, we fit the slope of the theoretical curves to its experimental value only. We do this separately for each element. We have minimized the sum of the square of discrepancies between the experimental and theoretical results for various combinations of the (α, p) parameters:

$$\Sigma^2 = \sum_{i=1}^{220} [\langle r^2 \rangle_i - \langle r^2_{exp} \rangle_i]^2 \ , \tag{14}$$

where

$$\langle r^2_{exp} \rangle_i = \delta \langle r^2_{exp} \rangle_i + \langle r^2_{av} \rangle_i \ . \tag{15}$$

Table 2 illustrates the results obtained for various sets of parameters.

TABLE 2

List of parameters from formula (13)

r_0	α	p	Σ^2	spherical nuclear radius
1.2	–	–	29.285	$R_0 = r_0 \cdot A^{1/3}$
1.2	0.203	1	2.655	$R_0 = 1.2[1 - \alpha\{\cdot(\frac{N-Z}{A})^p - (\frac{N-Z}{A})^p_\beta\}]A^{1/3}$
1.2	0.611	2	3.859	
1.23	0.609	2	4.107	$R_0 = r_0[1 - \alpha \cdot (\frac{N-Z}{A})^p]A^{1/3}$
1.25	0.2	1	2.795	

The first row in Table 2 corresponds to the traditional nuclear radius (1) for which the sum of the errors square, $\sum^2$, is about ten times larger than for any one of the new

formulas (13). The next two rows correspond to the case presented in Ref. [4] where the traditional $r_0 = 1.2$ fm radius was used and the value of

$$\left(\frac{N-Z}{A}\right)_\beta = \frac{0.4 \cdot A^2}{200 + A} \tag{15}$$

(Green's formula) was subtracted from the isotope factor in order to retain the old r_0 value which was found for nuclei along the line of β stability. Although the case of $\alpha = 0.203$ is the best one, nevertheless it is much more convenient to use the formula (13) for all the nuclei along and far-off the line of β stability. In such a case the last set of parameters ($r_0 = 1.25 fm, \alpha = 0.2, p = 1$) is the optimal one and we will choose it to check in the near future the ability to reproduce the nuclear masses.

It is necessary to stress that the values of the r_0 and α parameters do not change much when one increases the number of fitted nuclei. In Table 3 one can see that when taking the first region of the neutron-deficient and neutron-rich (ND + NR) nuclei into account (63 points) the best set of the α and r_0 parameters is almost the same as for all three regions of Sr – W, rare-earth and actinides, which combined give 220 points. Furthermore, the Σ^2 value remains more than 10 times smaller in comparison to the traditional formula (1). It means that formula (13) represents a more fundamental and general property of nuclear matter. The variance of the parameters in the formula (13) obtained when including new regions of nuclei into our investigation is presented in Table 3. The analysis was performed for the neutron-rich (NR), neutron-deficient (ND), rare-earth (RE) and actinides (AC) nuclei.

TABLE 3

Region	ND+NR	ND+NR+RE	ND+NR+RE+AC	
Number of isotopes	63	161	220	220
r_0	1.2	1.2	1.2	1.25
α	0.205	0.223	0.203	0.2
p	1	1	1	1

We now extend our present study to odd nuclei, although one does not expect much change in the set of the (r_0, α, p) parameters. The results are illustrated in Figures 9,

10, 11 -(a,b,c). Although, a fit of parameters in formula (13) was done simultaneously for all 220 nuclei, the results are presented separately for the sake of clarity of presentation. Every set shows a traditional liquid drop dependence of the MSR on the neutron number corresponding to the formula (4) (i.e. α=0, r_0=1.2 fm). Since the calculated results without the isospin dependence in R_0 are so different from the experimental data, it is impossible to present all the elements in the same figure. Therefore, there are two figures, labelled a and b, for every second element. The slopes of the theoretical MSR values (points) are much larger than the experimental ones (crosses). This effect can be observed for the neutron rich and deficient (Figs. 9a,b), the rare-earth nuclei (Figs. 10a,b) and the actinides (Fig. 11a,b). When the R_0 dependence on the neutron excess is changed to that of equation (13) the theoretical results become much better and the MSR slopes agree in an excellent way with the experimental data for all 220 nuclei as can be seen in Figs. 9c, 10c and 11c. There still remain some discrepancies for the lighter Sr isotopes and some of the heaviest nuclei, but one should keep in mind that these results were obtained using a macroscopic calculation only. When the new radius dependence on the neutron excess, equation (13), will be included into a more sophisticated dynamical macroscopic-microscopic model, the agreement should improve due to the inclusion of shell structure effects.

We will repeat our calculations, as described in Section 2, using a Woods–Saxon single particle potential. However first a new set of its parameters, which includes the new isospin dependence of the nuclear radius (equation 13), has to be found.

The new isospin dependence of the nuclear radius will influence all theoretical predictions for heavy-ion collisions, nuclear masses, Coulomb interaction in the drop and droplet models, and will probably simplify theoretical description of many effects connected with the neutron excess in a nucleus. At the present time when a lot of new experimental data on nuclei far from the line of β stability is being obtained, a better description of their radii is very important, and we are strongly convinced that the formula (13) reflects a true property of nuclear forces.

CONCLUSIONS

The following conclusions can be drawn from our investigation:

1. It is possible to obtain reasonable MSR isotope shifts and quadrupole moments for all even-even nuclei within a single microscopic dynamical model based on the Nilsson single particle potential, the Strutinsky potential energy and the generator coordinate method.
2. In general the collective variables, quadrupole ε and hexadecapole ε_4 deformations, are sufficient to describe nuclear deformations for all nuclei with the exception of:
   a) neutron-rich and -deficient nuclei with the neutron number close to the magic number 50, as these require a nonaxial γ variable, and
   b) the Ra – Th ($A \sim 220$) nuclei which have also the octupole deformation ε_3.
3. For every element the nuclear radius grows slower with the neutron number than $A^{1/3}$, and due to this fact the isospin term in $\hbar\omega_0$ oscillator frequency of the Nilsson potential should be absent or should be taken as very small.
4. The radius of the spherical nuclei should be described by the formula: $R_0 = 1.25 \cdot (1 - 0.2 \cdot \frac{N-Z}{A}) A^{1/3}\,\mathrm{fm}$, which enables one to get a good agreement between the macroscopic estimates of the MSR isotope shifts and the experimental data for all even-even nuclei. The sum of the square errors of $\delta\langle r^2\rangle_{A,A'}$ decreases by more than a factor 10 in comparison with that for the frequently used formula: $R_0 = 1.2\, A^{1/3}\, fm$.

Acknowledgements

We acknowledge valuable discussions with R. Hilton from Technical University in Munich and A. Blin from Coimbra University as well as the warm hospitality of the Theoretical Physics Group from C.R.N. in Strasbourg during the course of this work.

References

1. E.W.Otten,*Treatise on Heavy – Ion Science*, Vol. 8, ed. D. Allan Browley (1989) 517
2. S. Raman, C.H. Matarkey, W.T. Milner, C.W. Nestor,jr. P.H. Stelson, *Atomic Data and Nucl. Data Tables* **36/1** (1987) 1
3. B. Nerlo-Pomorska, K. Pomorski, B. Skorupska – Mach, *Nucl. Phys.* **A 562** (1993) 80
4. B. Nerlo-Pomorska, K. Pomorski, *Z. Phys.* **A 344** (1993) 359
5. P. Ring, P. Schuck: *"The Nuclear Many-Body Problem",* Springer-Verlag, 1980
6. T.Seo, *Z. Phys.* **A 324** (1986) 43
7. H.de Vries, C.W. de Jager, C.de Vries, *Atomic Data and Nucl. Data Tables* **36** (1987) 495
8. A. Bohr, B.R. Mottelson, *Nuclear Structure*, Vol. 2, N.Y. Benjamin, 1975
9. V.M. Strutinsky,*Nucl. Phys.* **A 95** (1967) 420
10. St. Piłat , K. Pomorski, A. Staszczak, *Z. Phys.***A 332** (1989) 259
11. P. Aufmuth, K. Heiling, S. Steudel, *Atomic Data and Nucl. Data Tables* **37** (1987) 455
12. P. Bonche, J. Dobaczewski, H. Flocard, P.H. Heenen, *Nucl. Phys.* **A 530** (1991) 149
13. W.D. Myers, K.-H. Schmidt, *Nucl. Phys.* **A410** (1983) 61

Figure Captions

1. Charge mean square radius isotope shifts $\delta\langle r^2\rangle^{A,A'}$ of light nuclei in fm^2 related to the A' mass number corresponding to the neutron numbers N marked on abscissa by the arrows. Points (•) denote theoretical microscopic results, while crosses (+) — the experimental values taken from Ref. [1].
2. Electric quadrupole moments of light nuclei. Points (•) denote theoretical microscopic dynamical results, crosses (+) — the experimental data.
3. The same as for Fig. 1 but for the neutron-rich and -deficient Sr – W nuclei. In the case when deformation energies of a prolate and an oblate energy minimum do not differ by more than 1 MeV, the results corresponding to the other energy minimum are presented as well.
4. The same as for Fig. 2 but for the neutron-rich and -deficient Sr – W nuclei. Open circles denote $\delta\langle r^2\rangle$ at another energy minimum (higher than the equilibrium one) in the cases when such results agree better with the experimental data.
5. The same as for Fig. 1 but for the rare-earth nuclei.
6. The same as for Fig. 2 but for the rare-earth nuclei.
7. The same as for Fig. 1 but for the actinides.
8. The same as for Fig. 2 but for the actinides.

9a,b. Macroscopic MSR $\langle r^2\rangle$ corresponding to the uniform charge distribution of the Sr – W nuclei in the microscopic equilibrium deformation obtained with the traditional $R_0 = 1.2 \cdot A^{1/3} fm$ nuclear radius. A discrepancy between the macroscopic (points) and experimental (crosses) results is so large that only every second element could be presented. Therefore there are two figures: a) for Sr–Ce, and b) for Zr–Ba. The experimental data is shifted perpendicularly to the N axis in order to cover optimally the theoretical and experimental curves.

9c. The same as for Figs. 9a and 9b but calculated with the radius $R_0 = 1.25 \cdot (1 - 0.2 \cdot \frac{N-Z}{A}) A^{1/3}$ fm.

10a,b. The same as for Figs. 9a,b but for the rare-earth nuclei.

10c. The same as for Fig. 9c but for rare-earth nuclei.

11a,b. The same as for Figs. 9a,b but for all even-even actinides known experimentally.

11c. The same as for Fig. 9c but for the actinides.

EXPLANATION OF TABLES

Table 4

The numerical results for the light nuclei $20 \leq Z \leq 48$, $Z \leq N \leq 48$ evaluated with the parameter set corresponding to the average mass number ($\mathrm{A}_{av} \sim 65$). The following notation is used:

Z	proton number
N	neutron number
A	mass number
ε^0	quadrupole equilibrium deformation
ε_4^0	hexadecapole equilibrium deformation
E_{def}	deformation energy in MeV: $E_{def} = V(\varepsilon^0, \varepsilon_4^0) - V(0,0)$
$< r^2 >$	theoretical estimate of the charge MSR in fm^2
$\delta < r^2 >^{A,A'}$	theoretical isotope shifts of the charge MSR: $\delta < r^2 >^{A,A'} = < r^2 >^{A} - < r^2 >^{A'}$
$\delta < r^2 >^{A,A'}_{exp}$	experimental value of the charge MSR isotope shifts in fm^2. The data written under [a] are taken from the paper: P. Aufmuth, K. Heiling, S. Steudel, *Atomic Data and Nucl. Data Tables* **37** (1987) 455. while the data written under [b] origins from the paper: E.W.Otten,*Treatise on Heavy – Ion Science*, Vol. 8, ed. D. Allan Browley (1989) 517
Q_2	electric quadrupole dynamical moments in b
Q_2^{exp}	experimental quadrupole moments in b taken from Ref. E.W.Otten,*Treatise on Heavy – Ion Science*, Vol. 8, ed. D. Allan Browley (1989) 517. The data written under [c] are taken from the paper: S. Raman, C.H. Matarkey, W.T. Milner, C.W. Nestor,jr. P.H. Stelson, *Atomic Data and Nucl. Data Tables* **36/1** (1987) 1

Table 5

The numerical results for the Sr – W even-even isotopes: $38 \leq Z \leq 74$, Z$\leq N \leq 74$, ($\mathrm{A}_{av} = 100$) evaluated in the oblate and prolate energy minima. Notation is the same as in Table 4. The data written under [d] are taken from the paper: H. Mach, F.K. Wohn, G. Molinar, K. Sistemich, J.C. Miehé, M. Moszyński, R.L. Gill, K. Krips, D.S. Brenner, Nucl Phys. **523** (1991) 197

Table 6

The numerical results for the rare-earth even-even nuclei: $52 \leq Z \leq 82$, $82 \leq N \leq 120$, ($\mathrm{A}_{av} = 165$) evaluated in the oblate and prolate energy minima.

Table 7

The numerical results for the actinides even-even isotopes with $82 \leq Z \leq 98$, $Z \leq N \leq 154$, ($\mathrm{A}_{av} = 217$).

This figure "fig1-3.png" is available in "png" format from:

http://arXiv.org/ps/nucl-th/9311026v1

This figure "fig5-4.png" is available in "png" format from:

http://arXiv.org/ps/nucl-th/9311026v1

This figure "fig1-26.png" is available in "png" format from:

http://arXiv.org/ps/nucl-th/9311026v1